

YGB - MPI PAPER 2 - QUESTION 1

EXPANDING IN THE "USUAL" MANNER

$$\begin{aligned}
 \left(x + \frac{2}{x}\right)^6 &= \binom{6}{0}(x)^6\left(\frac{2}{x}\right)^0 + \binom{6}{1}(x)^5\left(\frac{2}{x}\right)^1 + \binom{6}{2}(x)^4\left(\frac{2}{x}\right)^2 + \binom{6}{3}(x)^3\left(\frac{2}{x}\right)^3 \\
 &\quad + \binom{6}{4}(x)^2\left(\frac{2}{x}\right)^4 + \binom{6}{5}(x)^1\left(\frac{2}{x}\right)^5 + \binom{6}{6}(x)^0\left(\frac{2}{x}\right)^6 \\
 &= (1 \times x^6 \times 1) + (6 \times x^5 \times \frac{2}{x}) + (15 \times x^4 \times \frac{4}{x^2}) + (20 \times x^3 \times \frac{8}{x^3}) \\
 &\quad + (15 \times x^2 \times \frac{16}{x^4}) + (6 \times x \times \frac{32}{x^5}) + (1 \times 1 \times \frac{64}{x^6}) \\
 &= x^6 + 12x^4 + 60x^2 + 160 + \frac{240}{x^2} + \frac{192}{x^4} + \frac{64}{x^6}
 \end{aligned}$$

YGB - MPI PAPER 2 - QUESTION 2

a) SOLVING THE EQUATIONS SIMULTANEOUSLY

$$\begin{aligned} \left. \begin{aligned} y &= x^2 - 5x + 9 \\ y &= 5 \end{aligned} \right\} &\Rightarrow 5 = x^2 - 5x + 9 \\ &\Rightarrow 0 = x^2 - 5x + 4 \\ &\Rightarrow 0 = (x-1)(x-4) \\ &\Rightarrow x = \begin{matrix} 1 \\ 4 \end{matrix} \end{aligned}$$

$\therefore \underline{A(1,5) \text{ and } B(4,5)}$

b) LOOKING AT THE DIAGRAM BELOW



• $A_1 = 5 \times 3 = 15$

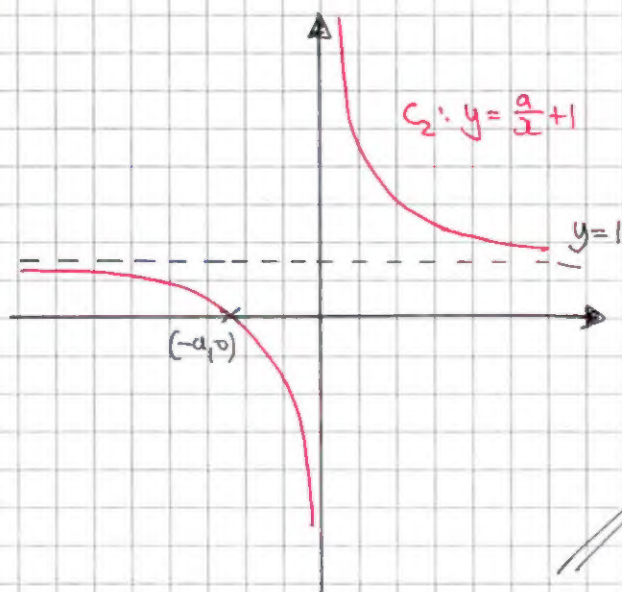
• $A_2 = \int_1^4 x^2 - 5x + 9 \, dx = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 + 9x \right]_1^4$
 $= \left(\frac{1}{3} \times 4^3 - \frac{5}{2} \times 4^2 + 9 \times 4 \right) - \left(\frac{1}{3} \times 1^3 - \frac{5}{2} \times 1^2 + 9 \times 1 \right)$
 $= \left(\frac{64}{3} - 40 + 36 \right) - \left(\frac{1}{3} - \frac{5}{2} + 9 \right) = \frac{52}{3} - \frac{41}{6} = \frac{21}{2}$

• REQUIRED AREA = $A_1 - A_2 = 15 - \frac{21}{2} = \underline{\underline{\frac{9}{2}}}$

1YGB - MPI PAPER 2 - QUESTION 3

- a) IT IS A TRANSLATION BY THE VECTOR $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, i.e. TRANSLATION "UPWARDS" BY ONE UNIT

b)



- NO y INTERCEPT
- $x=0$ & $y=1$ ARE THE TWO ASYMPTOTES
- $x=-a$ IS THE x INTERCEPT
SINCE $y=0$ GIVES
$$0 = \frac{a}{x} + 1$$
$$\frac{a}{x} = -1$$
$$-x = a$$
$$x = -a$$

c) SOLVING SIMULTANEOUS EQUATIONS

$$y = \frac{a}{x} + 1$$

$$y = x$$

I) USING $(-2, -2)$ IN THE FIRST EQUATION

$$-2 = \frac{a}{-2} + 1$$

$$4 = a - 2$$

$$a = 6$$

II) Now

$$\left. \begin{array}{l} y = \frac{6}{x} + 1 \\ y = x \end{array} \right\} \Rightarrow x = \frac{6}{x} + 1$$
$$\Rightarrow x^2 = 6 + x$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x+2)(x-3) = 0$$

POINT A

$$\therefore x = \begin{array}{l} -2 \\ 3 \end{array}$$

$\therefore B(3, 3)$

-1-

YGB - MPI PAPER 2 - QUESTION 4

Assertion: $m^2 - n^2 \neq 102$ if $m \in \mathbb{N}, n \in \mathbb{N}$

PROOF BY EXHAUSTION

REWRITE THE LHS AS A DIFFERENCE OF SQUARES

$$f(m, n) = m^2 - n^2 = (m+n)(m-n)$$

SUPPOSE THAT

(I) BOTH m, n ARE EVEN $\Rightarrow m+n$ AND $m-n$ WILL ALSO BE EVEN

$$\Rightarrow \begin{cases} m+n = 2\alpha \\ m-n = 2\beta \end{cases} \quad \alpha, \beta \in \mathbb{N}$$

$$\Rightarrow f(m, n) = (2\alpha)(2\beta) = 4\alpha\beta$$

$$\Rightarrow \underline{f(m, n) \text{ DIVIDES BY 4}}$$

BUT 102 DOES NOT

(II) BOTH m, n ARE ODD $\Rightarrow m+n$ AND $m-n$ WILL BE

\Rightarrow BY IDENTICAL ARGUMENT AS IN (I)
THIS IS NOT POSSIBLE

(III) IF m IS ODD & n IS EVEN (OR THE OTHER ROUND), THEN BOTH
 $m+n$ AND $m-n$ WILL BE ODD

$$\Rightarrow \begin{cases} m+n = 2\lambda+1 \\ m-n = 2\mu+1 \end{cases} \quad \lambda, \mu \in \mathbb{N}$$

$$\Rightarrow f(m, n) = (2\lambda+1)(2\mu+1)$$

1YGB - MA PAPER 2 - QUESTION 4

$$\Rightarrow f(m, n) = 2\lambda + 2\mu + 4\lambda\mu + 1$$

$$\Rightarrow f(m, n) = 2[2\lambda\mu + \lambda + \mu] + 1$$

$$\Rightarrow \underline{f(m, n) \text{ IS ODD BUT } 102 \text{ IS NOT}}$$

HENCE WE EXHAUSTED ALL THE POSSIBILITIES AND ALL OF
THE POSSIBLE SCENARIOS CANNOT PRODUCE 102

$$\therefore \underline{m^2 - n^2 \neq 102 \text{ IF } m \in \mathbb{N} \text{ \& } n \in \mathbb{N}}$$

IXGB - MPI PAPER 2 - QUESTION 5

a) COMPLETING THE SQUARE IN x & y

$$\Rightarrow x^2 + y^2 - 4x - 2y = 13$$

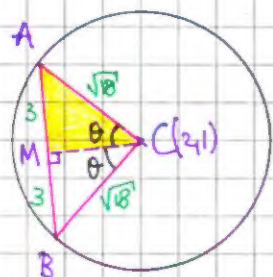
$$\Rightarrow x^2 - 4x + y^2 - 2y = 13$$

$$\Rightarrow (x-2)^2 - 4 + (y-1)^2 - 1 = 13$$

$$\Rightarrow (x-2)^2 + (y-1)^2 = 18$$

\therefore centre $C(2,1)$, radius $= \sqrt{18}$

b) LOOKING AT THE DIAGRAM BELOW



$$\sin \theta = \frac{3}{\sqrt{18}}$$

$$\sin \theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

$$\therefore \hat{ACB} = 2 \times 45^\circ$$

$$\hat{ACB} = 90^\circ$$

As required

OR

$$|AM|^2 + |MC|^2 = |AC|^2$$

$$3^2 + |MC|^2 = (\sqrt{18})^2$$

$$9 + |MC|^2 = 18$$

$$|MC|^2 = 9$$

$$|MC| = 3$$

$\therefore \triangle AMC$ IS RIGHT ANGLED AND ISOSCELES

$$\therefore \hat{ACM} = 45^\circ$$

$$\therefore \hat{ACB} = 90^\circ$$

c) SOLVING SIMULTANEOUSLY THE EQUATIONS OF THE CIRCLE & THE UNIT

$$\left. \begin{array}{l} x^2 + y^2 - 4x - 2y = 13 \\ y = k - 2 \end{array} \right\}$$

$$\Rightarrow x^2 + (k-2)^2 - 4x - 2(k-2) = 13$$

$$\Rightarrow x^2 + k^2 - 2kx + x^2 - 4x - 2k + 2 = 13$$

$$\Rightarrow 2x^2 - 2kx - 2x + k^2 - 2k - 13 = 0$$

$$\Rightarrow 2x^2 - 2x(k+1) + (k^2 - 2k - 13) = 0$$

$$\Rightarrow 2x^2 - 2(k+1)x + (k^2 - 2k - 13) = 0$$

As required

IYGB - MPI PAPER 2 - QUESTION 5

- d) IF THE LINE IS A TANGENT, THEN THE QUADRATIC IN x FOUND IN PART (C) MUST HAVE REPEATED ROOTS.

$$2x^2 - 2(k+1)x + (k^2 - 2k - 13) = 0$$

$$\begin{aligned} \underline{b^2 - 4ac = 0} \quad \text{with} \quad & \underline{a = 2} \\ & \underline{b = -2(k+1)} \\ & \underline{c = k^2 - 2k - 13} \end{aligned}$$

$$\Rightarrow [-2(k+1)]^2 - 4 \times 2 \times (k^2 - 2k - 13) = 0$$

$$\Rightarrow 4(k+1)^2 - 8(k^2 - 2k - 13) = 0 \quad \downarrow \div 4$$

$$\Rightarrow (k+1)^2 - 2(k^2 - 2k - 13) = 0$$

$$\Rightarrow k^2 + 2k + 1 - 2k^2 + 4k + 26 = 0$$

$$\Rightarrow -k^2 + 6k + 27 = 0$$

$$\Rightarrow k^2 - 6k - 27 = 0$$

$$\Rightarrow (k-9)(k+3) = 0$$



$$\Rightarrow \underline{k = \begin{matrix} 9 \\ -3 \end{matrix}} //$$

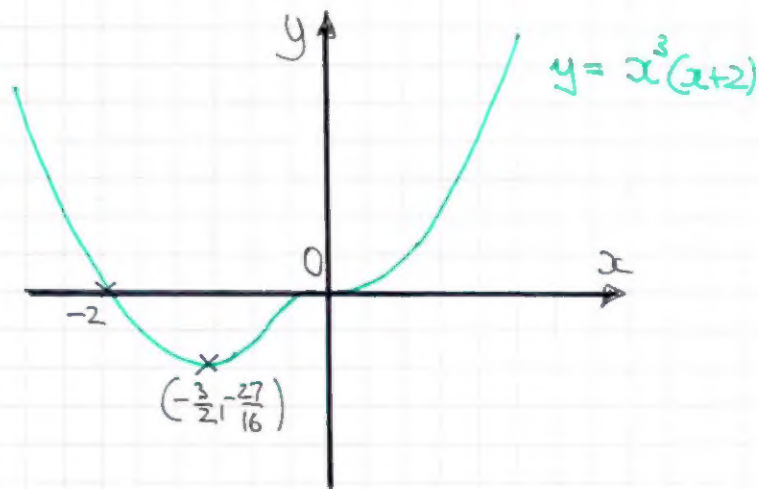
1YGB - MPI PAPER 2 - QUESTION 6

a)

$$y = x^3(x+2), \quad x \in \mathbb{R}$$

EXPAND AND START COLLECTING INFORMATION FOR THE SKETCH

- $y = x^4 + 2x^3 = x^3(x+2)$
- $(0,0)$,  or , it's a STATIONARY POINT OF INFLEXION
- $(-2,0)$, STATIONARY x INTERCEPT
- AS $x \rightarrow +\infty$, $y \sim x^4$, it GETS LARGE & POSITIVE
- AS $x \rightarrow -\infty$, $y \sim x^4$, it GETS LARGE & POSITIVE



- $\frac{dy}{dx} = 4x^3 + 6x^2 = 2x^2(2x+3)$

SOLVING FOR ZERO YIELDS $x = \begin{cases} 0 & \text{(POINT OF INFLECTION)} \\ -\frac{3}{2} & \text{(MINIMUM AS SEEN ON THE GRAPH)} \end{cases}$

$$y \Big|_{x=-\frac{3}{2}} = \left(-\frac{3}{2}\right)^3 \left(-\frac{3}{2} + 2\right) = -\frac{27}{8} \times \frac{1}{2} = -\frac{27}{16}$$

YGB - MPI PAPER 2 - QUESTION 6

b)

SET THE GRADIENT FUNCTION EQUAL TO 10

$$\Rightarrow 4x^3 + 6x^2 = 10$$

$$\Rightarrow 2x^3 + 3x^2 = 5$$

$$\Rightarrow 2x^3 + 3x^2 - 5 = 0$$

BY INSPECTION $x=1$ IS A SOLUTION - PROCEED
BY LONG DIVISION OR MANIPULATION

$$\Rightarrow 2x^2(x-1) + 5x(x-1) + 5(x-1) = 0$$

$$\Rightarrow (2x^2 + 5x + 5)(x-1) = 0$$

CHECK THE DISCRIMINANT OF THE QUADRATIC

$$\Rightarrow b^2 - 4ac = 5^2 - 4 \times 2 \times 5$$

$$= 25 - 40$$

$$= -15 < 0$$

\therefore NO MORE SOLUTIONS & ONLY POINT ON THE
CURVE IS THE POINT $P(1,3)$

- 1 -

NGB - UPI PAPER R - QUESTION 7

a) LOOKING AT THE DIAGRAM ON \hat{ACG}

$$\frac{|CG|}{|AG|} = \tan 60^\circ$$

$$\frac{60}{|AG|} = \sqrt{3}$$

$$|AG| = \frac{60}{\sqrt{3}}$$

$$|AG| = 20\sqrt{3}$$

BY PYTHAGORAS ON \hat{CGB}

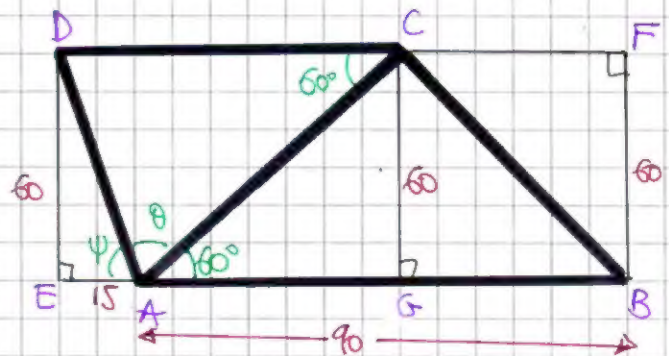
$$|CB|^2 = |CG|^2 + |GB|^2$$

$$|CB|^2 = 60^2 + (90 - |AG|)^2$$

$$|CB|^2 = 60^2 + (90 - 20\sqrt{3})^2$$

$$|CB|^2 = 6664.617093 \dots$$

$$|CB| \approx 81.6$$



q BY SIMPLE CONSIDERATIONS.

$$|CD| = |EA| + |AG|$$

$$|CD| = 15 + 20\sqrt{3}$$

$$|CD| \approx 49.6$$

b) FIRSTLY FIND ψ IN THE DIAGRAM

$$\tan \psi = \frac{|DE|}{|EA|}$$

$$\tan \psi = \frac{60}{15}$$

$$\psi = 75.9637^\circ \dots$$

$$\begin{aligned} \therefore \hat{DAC} = \theta &= 180 - 60 - \psi \\ &= 180 - 60 - 75.9637^\circ \\ &= 44.0^\circ \end{aligned}$$

1 d.p.

YGB - MPI PAGE 2 - QUESTION 8

MANIPULATE THE EQUATIONS, SO WE CAN "REMOVE" THE LOGS

$$\Rightarrow \log_2(x^2y) = 2$$

$$\Rightarrow \log_2(x^2y) = 2\log_2 2$$

$$\Rightarrow \log_2(x^2y) = \log_2 4$$

$$\Rightarrow x^2y = 4$$

$$\Rightarrow 11 + \frac{1}{2}\log_2 y = 3\log_2 x$$

$$\Rightarrow 22 + \log_2 y = 6\log_2 x^3$$

$$\Rightarrow 22\log_2 2 + \log_2 y = \log_2 x^6$$

$$\Rightarrow \log_2 2^{22} + \log_2 y = \log_2 x^6$$

$$\Rightarrow \log(2^{22} \times y) = \log_2 x^6$$

$$\Rightarrow y \times 2^{22} = x^6$$

SOLVING BY DIVISION

$$\left. \begin{array}{l} y \times 2^{22} = x^6 \\ x^2 y = 4 \end{array} \right\} \Rightarrow$$

$$\frac{2^{22}}{x^2} = \frac{x^6}{4}$$

$$\Rightarrow 4 \times 2^{22} = x^8$$

$$\Rightarrow 2^{24} = x^8$$

$$\Rightarrow (2^3)^8 = x^8$$

$$\Rightarrow x^8 = 8^8$$

$$\Rightarrow x = 8$$

q using $x^2y = 4$

$$\Rightarrow 8^2 \times y = 4$$

$$\Rightarrow 64y = 4$$

$$\Rightarrow y = \frac{1}{16}$$

1YGB - MPI PAPER 2 - QUESTION 8

ALTERNATIVE METHOD/APPROACH

STARTING WITH THE EQUATION & PROCEED AS FOLLOWS

$$\left. \begin{array}{l} \log_2(x^2y) = 2 \\ 11 + \frac{1}{2}\log_2 y = 3\log_2 x \end{array} \right\} \Rightarrow \left. \begin{array}{l} \log_2 x^2 + \log_2 y = 2 \\ 11 + \frac{1}{2}\log_2 y = 3\log_2 x \end{array} \right\} \Rightarrow$$

$$\left. \begin{array}{l} 2\log_2 x + \log_2 y = 2 \\ 11 + \frac{1}{2}\log_2 y = 3\log_2 x \end{array} \right\} \Rightarrow \begin{array}{l} 2X + Y = 2 \\ 11 + \frac{1}{2}Y = 3X \quad \times(-2) \end{array}$$

$$\left. \begin{array}{l} 2X + Y = 2 \\ -22 + Y = -6X \end{array} \right\} \Rightarrow \begin{array}{l} 2X - 22 = 2 - 6X \\ 8X = 24 \end{array}$$

$$\Rightarrow \underline{X = 3}$$

$$\Rightarrow \log_2 x = 3$$

$$\Rightarrow \log_2 x = 3\log_2 2 = \log_2 8$$

$$\Rightarrow \underline{x = 8}$$

$$\& \quad 2X + Y = 2$$

$$6 + Y = 2$$

$$\underline{Y = -4}$$

$$\Rightarrow \log_2 y = -4$$

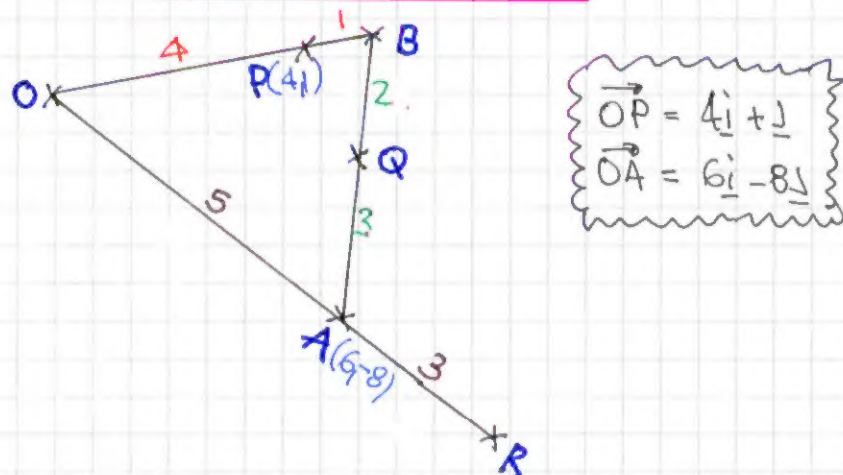
$$\Rightarrow \log_2 y = -4\log_2 2 = \log_2 2^{-4}$$

$$\Rightarrow y = 2^{-4}$$

$$\Rightarrow \underline{y = \frac{1}{16}}$$

1YGB - MPI PAPER 2 - QUESTION 9

a) LOOKING AT THE DIAGRAM



- $\vec{OB} = \frac{5}{4} \vec{OP} = \frac{5}{4}(4\mathbf{i} + \mathbf{j}) = 5\mathbf{i} + \frac{5}{4}\mathbf{j}$
- $\vec{BA} = \vec{BO} + \vec{OA} = -5\mathbf{i} - \frac{5}{4}\mathbf{j} + 6\mathbf{i} - 8\mathbf{j} = \mathbf{i} - \frac{37}{4}\mathbf{j}$
- $\vec{BQ} = \frac{2}{5} \vec{BA} = \frac{2}{5}(\mathbf{i} - \frac{37}{4}\mathbf{j}) = \frac{2}{5}\mathbf{i} - \frac{37}{10}\mathbf{j}$
- $\vec{OQ} = \vec{OB} + \vec{BQ} = 5\mathbf{i} + \frac{5}{4}\mathbf{j} + \frac{2}{5}\mathbf{i} - \frac{37}{10}\mathbf{j}$
 $= \frac{27}{5}\mathbf{i} - \frac{49}{20}\mathbf{j}$

$\therefore Q(5.4, -2.45)$

-1-

b) COMPARE THE VECTORS \vec{PQ} & \vec{QR}

- $\vec{PQ} = \vec{OQ} - \vec{OP}$ (USING POSITION VECTORS)
 $= (\frac{27}{5}\mathbf{i} - \frac{49}{20}\mathbf{j}) - (4\mathbf{i} + \mathbf{j}) = \frac{7}{5}\mathbf{i} - \frac{69}{20}\mathbf{j}$
- $\vec{QR} = \vec{OR} - \vec{OQ} = \frac{8}{5}\vec{OA} - \vec{OQ}$
 $= \frac{8}{5}(6\mathbf{i} - 8\mathbf{j}) - (\frac{27}{5}\mathbf{i} - \frac{49}{20}\mathbf{j})$
 $= \frac{21}{5}\mathbf{i} - \frac{207}{20}\mathbf{j}$

c) HENCE WE HAVE

- $\vec{PQ} = \frac{7}{5}\mathbf{i} - \frac{69}{20}\mathbf{j} = \frac{1}{20}(28\mathbf{i} - 69\mathbf{j})$
- $\vec{QR} = \frac{21}{5}\mathbf{i} - \frac{207}{20}\mathbf{j} = \frac{3}{20}(28\mathbf{i} - 69\mathbf{j})$

AS \vec{PQ} & \vec{QR} ARE IN THE DIRECTION OF THE SAME VECTOR $(28\mathbf{i} - 69\mathbf{j})$ AND SHARE THE POINT Q, THE POINTS P, Q & R ARE COLLINEAR WITH $|\vec{PQ}| : |\vec{QR}| = 1:3$

1YGB - MPI PAPER 2 - QUESTION 10

a) LOOKING AT THE DIAGRAM

● GRADIENT AB = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 10}{9 - (-3)} = \frac{-4}{12} = -\frac{1}{3}$

● GRADIENT OF $L_1 = +3$

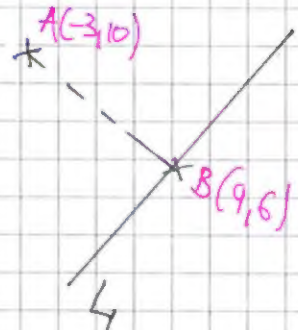
● EQUATION OF L_1 , WITH $m = 3$, PASSING THROUGH $(9, 6)$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 6 = 3(x - 9)$$

$$\Rightarrow y - 6 = 3x - 27$$

$$\Rightarrow y = 3x - 21$$



b) EITHER USE STANDARD FORM

OR

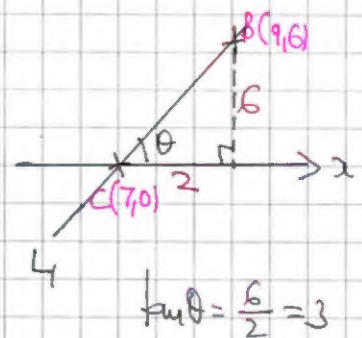
$$y = mx + c \text{ with } m = \tan \theta$$

$$\therefore \tan \theta = 3$$

FIND C FIRST

$$\text{when } y = 0 \quad x = 7$$

$$\therefore \underline{C(7, 0)}$$



c) GRADIENT OF AB = $-\frac{1}{3}$ (PART a)

EQUATION OF L_2 WITH $m = -\frac{1}{3}$ & $C(7, 0)$

$$\Rightarrow y - y_0 = m(x - x_0)$$

$$\Rightarrow y - 0 = -\frac{1}{3}(x - 7)$$

$$\Rightarrow 3y = -(x - 7)$$

$$\Rightarrow 3y = -x + 7$$

$$\Rightarrow x + 3y = 7$$

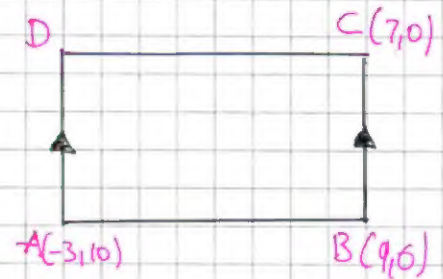
LYGB - MPI PAPER 7 - QUESTION 10

d) LOOKING AT THE DIAGRAM (NOT TO SCALE)

USING "VECTOR" CONCEPTS

$$\text{FROM B TO C} \Rightarrow \begin{pmatrix} 7-9 \\ 0-6 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$$

$$\text{FROM A TO D} \Rightarrow \begin{pmatrix} -3 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ -6 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$$



$$\therefore \underline{D(-5, 4)}$$

AS REQUIRED

e)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(-3-9)^2 + (10-6)^2} = \sqrt{144 + 16} = \sqrt{160} = 4\sqrt{10}$$

$$|BC| = \sqrt{(7-9)^2 + (0-6)^2} = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$$

$$\therefore \text{AREA} = |AB| |BC| = 4\sqrt{10} \times 2\sqrt{10} = 8 \times 10 = \underline{80}$$

- 1 -

1YGB - MPI PAPER 2 - QUESTION 11

FIRST WRITE THE EQUATION IN $\sin\theta$

$$y = 6 - 4\sin\theta - \cos^2\theta$$

$$y = 6 - 4\sin\theta - (1 - \sin^2\theta)$$

$$y = 5 - 4\sin\theta + \sin^2\theta$$

BY INSPECTION, LOOKING AT THE SINE GRAPH, AND NOTING THAT $\sin 270^\circ = -1$

$$y_{\max} = 5 - 4(-1) + (-1)^2 = 10$$

$$\therefore \underline{B(270^\circ, 10)}$$

COMPLETING THE SQUARE IN $\sin\theta$

$$y = \sin^2\theta - 4\sin\theta + 5$$

$$y = (\sin\theta - 2)^2 + 1$$

BUT $\sin\theta \neq 2$, SO MINIMUM WILL BE ACHIEVED WITH $\sin\theta = +1$

$$y_{\min} = (+1 - 2)^2 + 1 = 1 + 1 = 2$$

$$\therefore \underline{A(90^\circ, 2)}$$